

ADJOINTCEPTION

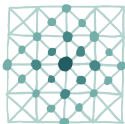
OR

HOW INTEGRATION BY PARTS MAY HELP
GEOLOGISTS TO X-RAY STONES

TAMME CLAUS (AGM)

PI: Manuel Torrilhon (AGM)

Coworkers: Pawrau Achuda (GFE), Silvia Richter (GFE)

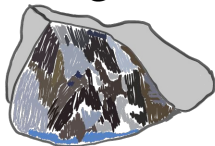


APPLIED AND
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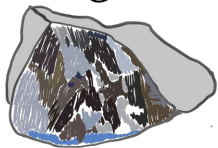
SETTING THE STAGE - PHYSICAL MOTIVATION

① FIND STONE

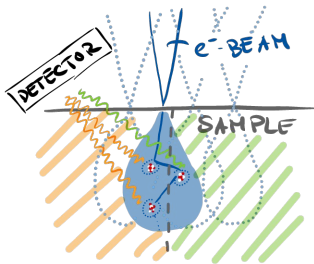


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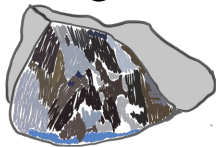


② PUT UNDER MICROSCOPE

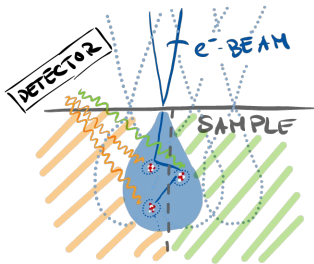


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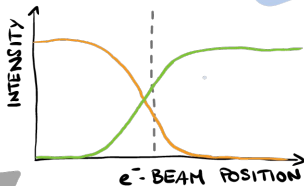
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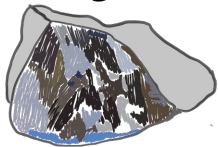


③ MEASURE k

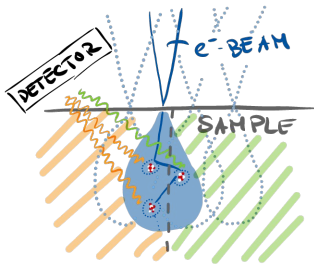


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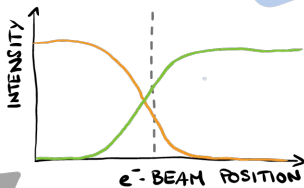
① FIND STONE



② PUT UNDER MICROSCOPE



③ MEASURE K



④ RECONSTRUCT ρ



SETTING THE STAGE - MATHEMATICAL TOOLS

INVERSE PROBLEM

e^- -transport
x-ray generation
x-ray attenuation

measurement

$$\min_{g \in \mathbb{R}} \|k(g) - k^{\text{exp}}\| = \delta(g)$$

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GRADIENT-BASED MINIMIZATION

$$g_{i+1} = g_i - \alpha \nabla_g \delta(g)$$

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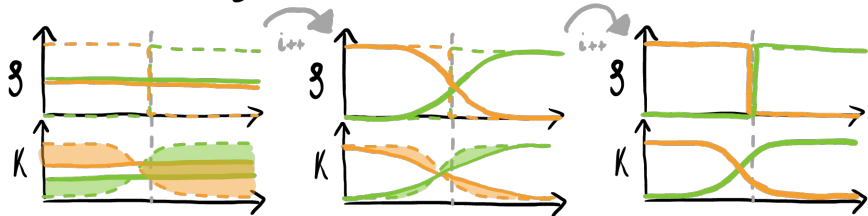
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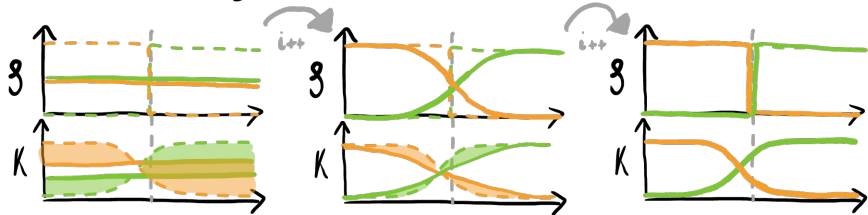
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$$\min_{g \in \mathbb{R}} \|k(g) - k^{\text{exp}}\| = \delta(g)$$

GRADIENT-BASED MINIMIZATION

$$g_{i+1} = g_i - \alpha \nabla_g J(g)$$

VERY EXPENSIVE TO COMPUTE
 (# mat. param \times # beam pos \times PDE'S)



ADJOINTS - TO REDUCE COMPUTATIONAL COMPLEXITY

Def: ADJOINT

$$\langle y, Ax \rangle_Y = \langle A^* y, x \rangle_X \quad \forall x \in X, y \in Y$$

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→ We can transform a numerical code

```
for  $i=1..I$  do
   $v_i = Ax_i$  ← EXPENSIVE
  for  $j=1..J$  do
     $\Sigma_{ij} = \langle y_j, v_i \rangle_y$ 
```

→ scales in I

⇔

```
for  $j=1..J$  do
   $v_j^* = A^*y_j$  ← EXPENSIVE
  for  $i=1..I$  do
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→ scales in J

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ADJOINTCEPTION

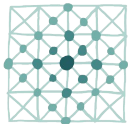
WE DO THIS TWICE!

or
HOW INTEGRATION BY PARTS MAY HELP
GEOLOGISTS TO X-RAY STONES

$\langle y, \partial_t x \rangle_y = \langle \partial_t y, x \rangle_x$

REFERENCES

- [1] U. Scholz. (2023) HOW TO DRAW YOUR PRESENTATION SLIDES. Dr. ref. wat night
- [2] J. Binger (2022). THREE-DIMENSIONAL MODELING OF X-RAY EMISSION IN ELECTRON PROBE MICROANALYSIS BASED ON DETERMINISTIC TRANSPORT EQUATIONS. RWTH Aachen University
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- [5] R-E Resnik. (2006). A REVIEW OF THE ADJOINT-STATE METHOD FOR COMPUTING THE GRADIENT OF A FUNCTIONAL WITH GEOPHYSICAL APPLICATIONS. Geophys J. Int. Vol 167 issue 2 pp. 495-503



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