Efficient Computation of K-Ratio Profiles in EPMA Using Adjoint Electron Transport

Tamme Claus[1] - claus@acom.rwth-aachen.de
Gaurav Achuda[2], Silvia Richter[2], Manuel Torrilhon[1]

[1] ACoM, Applied and Computational Mathematics, RWTH Aachen

[2] GFE, Central Facility for Electron Microscopy, RWTH Aachen

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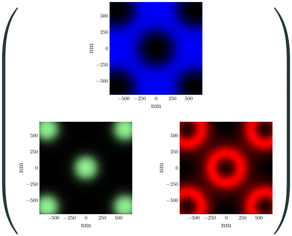
GFE Central Facility for Electron Microscopy | Applied and Computational Mathematics | RWTH AACHEN UNIVERSITY

Adjoint Electron Transport in EPMA

***Vision**: an efficient computational model for EPMA*

Current Approach:

- define a model 🔬(beam) : {beams} → x-ray intens. (e.g. MC or determ.)
- loop model 🔬(·) over beam positions → expensive



$$= \begin{pmatrix} 🔬(\text{beam}) & 🔬(\text{beam}) & \dots \\ 🔬(\text{beam}) & \ddots & \end{pmatrix}$$

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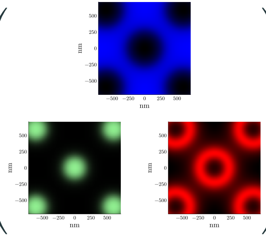


= (🔬( ) 🔬( ) … ; 🔬( ) ⋱)

If the model 🔬(·) is *linear (and bounded)* w.r.t. the beam, then
→ Riesz Representation: *there is a such that* 🔬( ) $= \langle\ ,\ \rangle_{\{\}}$.

Adjoint Electron Transport in EPMA

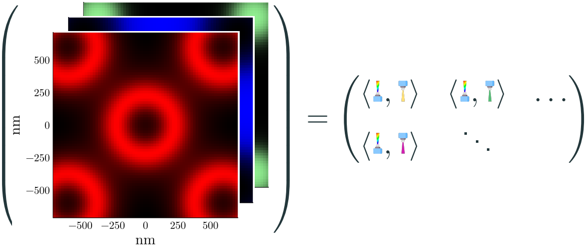
Riesz Representation: *there is a* *such that* $(\ ) = \langle\ ,\ \rangle_{\{\ \}}$.

Adjoint Electron Transport in EPMA

Riesz Representation: *there is a* · *such that* $(\cdot) = \langle \cdot, \cdot \rangle_{\{\cdot\}}$.

Adjoint Approach:

- compute $= {}^{T}(\blacksquare)$ [→ cost comparable to $(\cdot)$]
- loop $\langle \cdot, \cdot \rangle$ over beam positions → much cheaper



→ also useful for e.g. 'material derivatives' in inverse modeling

→ a similar concept accelerates 'training' in AI

[1] Halbleib & Morel (1980). Adjoint Monte Carlo electron transport in the continuous-slowing-down approximation. Journal of Computational Physics, 34(2), 211-230.

[2] Kuridan (2023). The Adjoint Transport Equation—The Equation of Neutron Importance. In: Neutron Transport. Graduate Texts in Physics. Springer, Cham.

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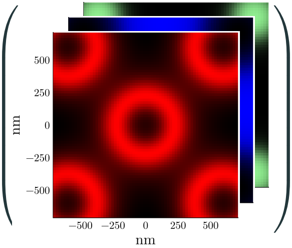

$= \begin{pmatrix} \langle\,,\,\rangle & \langle\,,\,\rangle & \cdots \\ \langle\,,\,\rangle & \ddots & \end{pmatrix}$

$70 \times 70 \times 198\text{s} \approx 11\text{days}\,5\text{h}$

$3 \times 198\text{s} \approx 10\text{min}$

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